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MEMORANDUM REPORT NO. 1538
FEBRUARY 1964

A METHOD FOR ESTIMATING DANGER AREAS DUE TO RICOCHETING PROJECTILES

D. J. Dunn
W. D. Dotson, Jr.

RDT & E Project No. 1M523801A287

BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND

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ERRATA SHEET

for

Ballistic Research Laboratories Memorandum Report No. 1538 entitled "A Method for Estimating Danger Areas Due to Richocheting Projectiles" by D. J. Dunne and W. D. Dotson, Jr. dated February 1964

Page 14, 8th line from bottom of page should read:
"of the richochet variables $\vec{V}$ and ϕ are used to compute the trajectory coordinates"

Page 14, 5th line from bottom of page should read:
"the trajectory coordinates fill a 3 dimensional volume whose envelope defines the"

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W. D. Dotson Jr.

Terminal Ballistics Laboratory
Computing Laboratory

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ABERDEEN PROVING GROUND, MARYLAND

B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

MEMORANDUM REPORT NO. 1536

DJDunn/WGDotson, Jr./jk
Aberdeen Proving Ground, Md.
February 1964

A METHOD FOR ESTIMATING DANGER AREAS DUE TO RICOCHETING PROJECTILES

ABSTRACT

The ricochet studies of Birkhoff and Hitchcock are summarized. A theory is developed for ricochet of projectiles off inclined planes, and this theory is applied to obtain a procedure for the estimation of danger areas due to ricocheting projectiles.

I. INTRODUCTION

In several instances over the past few years it has been necessary to estimate the danger area resulting from the ricochet of projectiles off various surfaces during outdoor training exercises or testing operations. Of particular interest was the lateral extent of the danger zone. To supply these estimates, an analytical procedure was worked out and is described in the present report.

Based upon a large collection of experimental data on ricochet off land surfaces, the following empirical formulas were published by G. Birkhoff[1] in 1945 (BRL Report No. 535):

$$\frac{|\vec{v}|}{|\vec{u}|} = 1.0 - 0.5 \frac{\theta}{\theta_c} \tag{1}$$

$$\frac{\phi}{\theta} = 2.5 - 1.5 \frac{\theta}{\theta_c} \tag{2}$$

Where (see Figure 1): θ = angle of impact, $|\vec{u}|$ = magnitude of velocity of impact, ϕ = angle of ricochet, $|\vec{v}|$ = magnitude of velocity of ricochet, and θ_c = the <u>critical angle</u> for the projectile/soil (or impact material) combination and is defined as that angle of impact above which ricochet occurs less than half the time.

In 1947, H. P. Hitchcock[2] published a large collection of additional experimental data on ricochet off land surfaces (BRL Report No. 629). Considering these additional data, D. J. Dunn obtained the following <u>modified Birkhoff formulas</u>:

$$\frac{|\vec{v}|}{|\vec{u}|} = 1.0 - 0.4 \frac{\theta}{\theta_c} \quad (\theta < \theta_c) \tag{3}$$

$$= 0 \quad (\theta \geq \theta_c)$$

$$\frac{\phi}{\theta} = 3.5 - 2.8 \frac{\theta}{\theta_c} \quad (\theta < \theta_c) \tag{4}$$

Graphs of Equations (3) and (4) are presented in Figure 2. These modified Birkhoff formulas are currently used at BRL for computation of velocity and angle of ricochet from velocity and angle of impact. The critical angle θ_c is a function of the nose shape, impact velocity, hardness of the projectile nose and hardness of the ricochet surface. The value of θ_c is obtained by searching through the ricochet literature for an experimental value corresponding to the appropriate

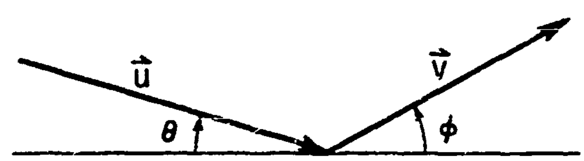

FIGURE 1.

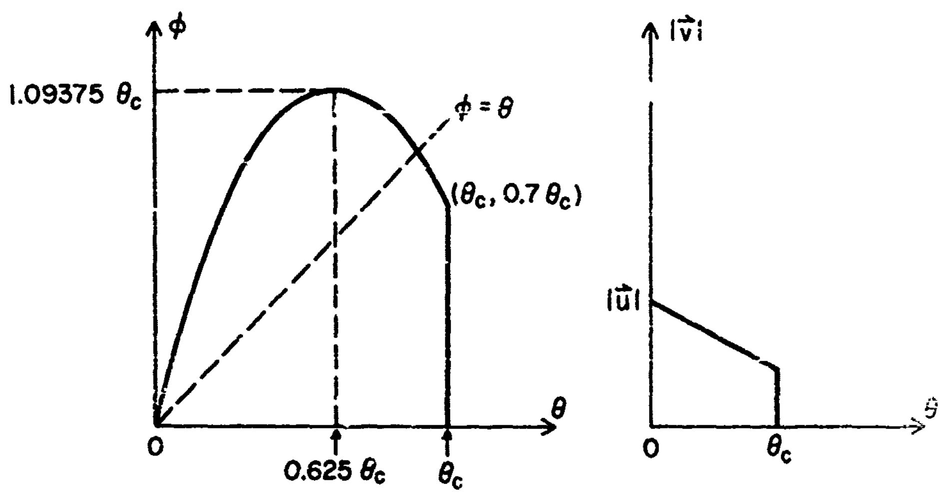

FIGURE 2

set of parameters. Notice in Equations (3) and (4), or in Figure 2, that both $|\vec{v}|$ and ϕ increase with an increase in θ_c -- assuming given values of the impact conditions $|\vec{u}|$ and θ. Consequently, for computation of danger zones, θ_c should be chosen as large as realism permits in order to avoid underestimating the size of the danger zone.

II. THEORY OF RICOCHET OFF INCLINED PLANES

Consider the impact of a given projectile on an inclined plane composed of a given material. We assume that the critical angle, θ_c, for the projectile/impact material combination has been determined (or appropriately chosen). The point of impact is taken as the origin (0, 0, 0) of an xyz rectangular coordinate system, Figure 3. The y axis is an extension of the radius-vector from the center of the earth to the origin (0, 0, 0), and the xz plane is perpendicular to the y axis. The x axis is chosen so that the impact velocity, $\vec{u}$, of the projectile is contained in the xy plane; and the z axis is then chosen so that the xyz system will be right-handed. Unit vectors along the x, y, and z axes are denoted by $\vec{i}$, $\vec{j}$, $\vec{k}$, respectively. Hence, we have

$$\vec{u} = u_x\vec{i} + u_y\vec{j} \tag{5}$$

$$|\vec{u}| = \sqrt{u_x^2 + u_y^2} \tag{6}$$

We assume that u_x and u_y are known from the output of a two-dimensional trajectory computation terminating at (0, 0, 0). Hence, $|\vec{u}|$ is also known. The orientation of the inclined plane is specified by a unit vector $\vec{n}$ perpendicular to the inclined plane at (0, 0, 0). We have

$$\vec{n} = n_x\vec{i} + n_y\vec{j} + n_z\vec{k} \tag{7}$$

$$|\vec{n}| = \sqrt{n_x^2 + n_y^2 + n_z^2} = 1 \tag{8}$$

The two components n_x and n_y will be arbitrarily specified, and n_z will then be determined, except for sign, from Equation (8). Adopting the convention that $n_z \geq 0$ (which amounts to considering half of a symmetrical distribution of possible orientations of the inclined plane -- i.e. symmetrical about the xy plane) we have

$$n_z = \sqrt{1 - n_x^2 - n_y^2} \tag{9}$$

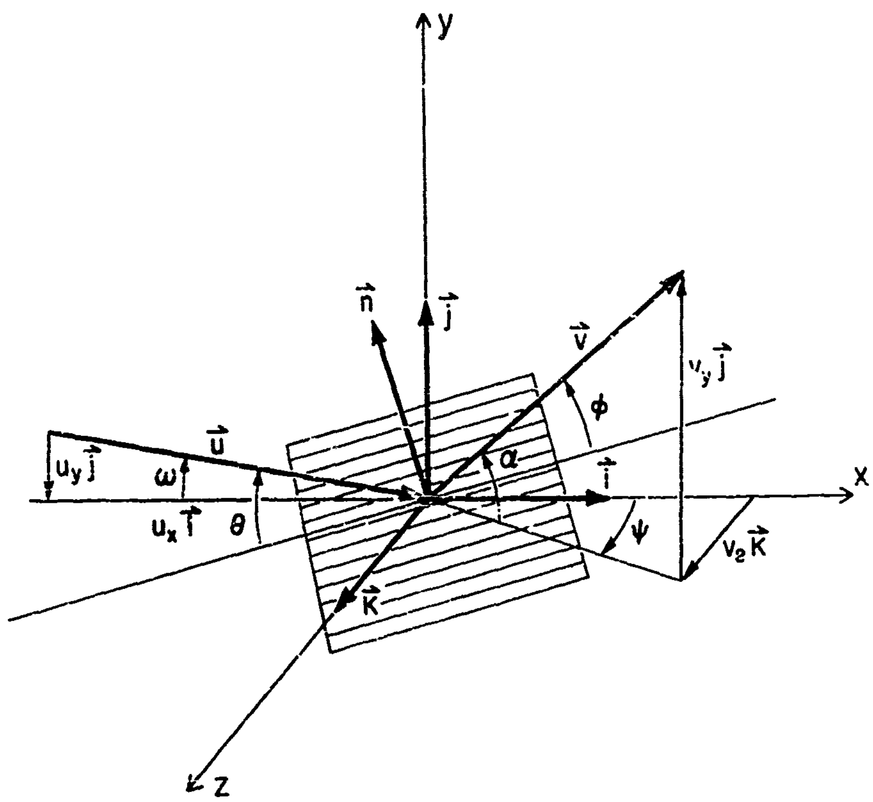
y
$\vec{n}$
$\vec{j}$
$\vec{v}$
$v_y \vec{j}$
ϕ
$\vec{u}$
$u_y \vec{j}$
ω
α
$\vec{i}$
X
$u_x \vec{i}$
θ
ψ
$V_z \vec{K}$
$\vec{K}$
Z

FIGURE 3.

The orientation of the inclined plane can, of course, be varied by varying n_x and n_y. For any particular orientation, our known quantities are u_x, u_y (and hence $|\vec{u}|$), n_x, n_y, n_z, and θ_c. In terms of these known quantities, we must calculate (1) the magnitude, $|\vec{v}|$, of the velocity $\vec{v}$ of ricochet, (2) the angle α that the vector $\vec{v}$ makes with the xz plane, and (3) the angle ψ that the projection of $\vec{v}$ on the xz plane makes with the x axis (see Figure 3). The initial conditions (i.e. initial velocity and angle of elevation) for the ricochet trajectory are $|\vec{v}|$ and α, and the range of the ricochet trajectory must be plotted so as to form the angle ψ with the x axis.

We will begin by determining the angle of impact, θ, that the vector $\vec{u}$ makes with the inclined plane (see Figure 3). We have (see Figure 4):

$$\begin{aligned} \vec{u} \cdot \vec{n} &= |\vec{u}|\ |\vec{n}|\ \cos(90^\circ + \theta) \\ &= |\vec{u}|\ (-\sin\theta) \end{aligned} \tag{10}$$

But, since $\vec{u} = u_x\vec{i} + u_y\vec{j}$ and since $\vec{n} = n_x\vec{i} + n_y\vec{j} + n_z\vec{k}$, we have

$$\vec{u} \cdot \vec{n} = u_x n_x + u_y n_y \tag{11}$$

Equations (10) and (11) yield the following solution for θ in terms of the known quantities u_x, u_y, n_x, n_y, $|\vec{u}|$:

$$\theta = \sin^{-1}\left\{\frac{-(u_x n_x + u_y n_y)}{|\vec{u}|}\right\} \tag{12}$$

In general we have $u_x > 0$, $n_x < 0$, $u_y < 0$, $n_y > 0$, so that θ will generally turn out to be a positive angle.

We now know $|\vec{u}|$, θ, and θ_c ; and so we can compute $|\vec{v}|$ and ϕ from the modified Birkhoff formulae, Equations (3) and (4). Of course, this ϕ is the angle that the vector $\vec{v}$ makes with the inclined plane. Hence, we can now add $|\vec{v}|$ and ϕ to our list of known quantities. To determine the angles α and ψ, we need to find the components of the vector $\vec{v}$ in the x, y, and z directions. We have

$$\vec{v} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k} \tag{13}$$

and so

$$v_x^2 + v_y^2 + v_z^2 = |\vec{v}|^2 \qquad (14)$$

Since $|\vec{v}|^2$ is known, Equation (14) is one condition for the determination of the unknowns v_x, v_y, v_z. We need two more conditions. We will assume that the vector $\vec{v}$ lies in the plane determined by $\vec{u}$ and $\vec{n}$ (see Figure 4). This assumption can be stated

$$(\vec{u} \times \vec{n}) \cdot \vec{v} = 0 \qquad (15)$$

Since

$$\vec{u} \times \vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_x & u_y & 0 \\ n_x & n_y & n_z \end{vmatrix}$$

and since $\vec{v}$ is given by Equation (13), Equation (15) reduces to

$$v_x n_z u_y - v_y n_z u_x + v_z (u_x n_y - u_y n_x) = 0 \qquad (16)$$

Referring to Figure 4, we have

$$\vec{u} \cdot \vec{v} = |\vec{u}| \; |\vec{v}| \; \cos(\theta + \phi) \qquad (17)$$

and so, since $\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y$, we have from Equation (17)

$$v_x u_x + v_y u_y = |\vec{u}| \; |\vec{v}| \; \cos(\theta + \phi) \qquad (18)$$

The only unknowns in Equations (16) and (18) are v_x, v_y, v_z; and (16) and (18) are linear in v_x, v_y, v_z. Hence, it is easy to solve (16) and (18) simultaneously for v_x and v_y in terms of v_z and known quantities. The results are as follows:

$$v_x = \frac{u_y v_z (u_y n_x - u_x n_y) + u_x n_z \; |\vec{u}| \; |\vec{v}| \; \cos(\theta + \phi)}{n_z \; |\vec{u}|^2} \qquad (19)$$

$$v_y = \frac{n_z u_y \; |\vec{u}| \; |\vec{v}| \; \cos(\theta + \phi) - u_x v_z (u_y n_x - u_x n_y)}{n_z \; |\vec{u}|^2} \qquad (20)$$

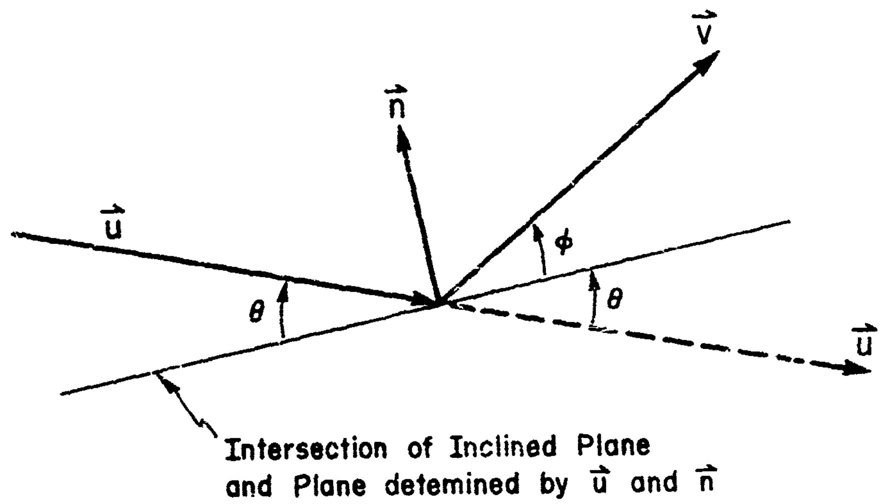
v
n
u
φ
θ
θ
u
Intersection of Inclined Plane
and Plane detemined by u and n

FIGURE 4.

Substituting Equations (19) and (20) for v_x and v_y in Equation (14), we obtain an equation in which the only unknown is v_z. This equation is easily solved to give

$$v_z = \frac{|\vec{v}| \sin(\theta + \phi)}{\sqrt{1 + \left(\frac{u_y n_x - u_x n_y}{n_z |\vec{u}|}\right)^2}} \tag{21}$$

v_x and v_y can now be computed by substituting this solution for v_z into Equatio (19) and (20). Finally, we have (see Figure 3):

$$\alpha = \sin^{-1} (v_y / |\vec{v}|) \tag{22}$$

$$\psi = \tan^{-1} (v_z / v_x) \tag{23}$$

III. ESTIMATION OF DANGER AREAS

The theory developed in Section II can be applied to estimate danger areas due to ricocheting projectiles. For the particular projectile, gun and line of fire under consideration, the velocity of impact $|\vec{u}|$ and angle of fall are known or can be computed for any range, R. Consider a particular range, R_1. We suppose the projectile strikes an inclined plane at R_1 . As the orientation of the inclined plane is varied, by varying the values of n_x and n_y , the various ricochet ranges are determined by means of the theory of Section II and the rest coordinates after ricochet are plotted in the x-z plane. This procedur is repeated for other pertinent values of R, say R_2, R_3, ...R_n , and for each range the set of rest coordinates is plotted. An envelope of the family of set points is sketched in to form the boundary of the danger area. In the interest of safety, it is generally assumed that the projectile flys just as well after ricochet as it did before ricochet -- i.e. the same ballistics are used to run the ricochet trajectories as were used to run the original trajectories. One exception to this is that windshields are assumed to break off after impact on hard surfaces. In any case, however, it is assumed that the flight of the projectile after impact is stable.

An example of a danger area, constructed according to the above procedure, is given in Figure 5.

IV. VERTICAL DANGER AREA

In certain cases a training area may be located in the vicinity of an airfield and training exercises may be in progress while aircraft are taking off or landing. It is of interest to determine the vertical area dangerous to the aircraft because of ricocheting projectiles. This problem is a special case of the lateral ricochet problem already described except for the following specializations. The values of the ricochet variables $\vec{v}$ and ϕ are used to compute the zenith coordinates, instead of the rest coordinates, after ricochet. As before, the coordinates are computed for each location and orientation of the ricochet plane. The totality of the zenith coordinates fill a 3 dimensional volume whose envelope defines the boundary of the vertical danger area. The boundary is now a surface instead of a line. The section of the envelope in the vertical plane which contains the line of fire is ordinarily of most interest since this section includes the highest zenith points.

D. J. DUNN

W. G. DOTSON, JR.

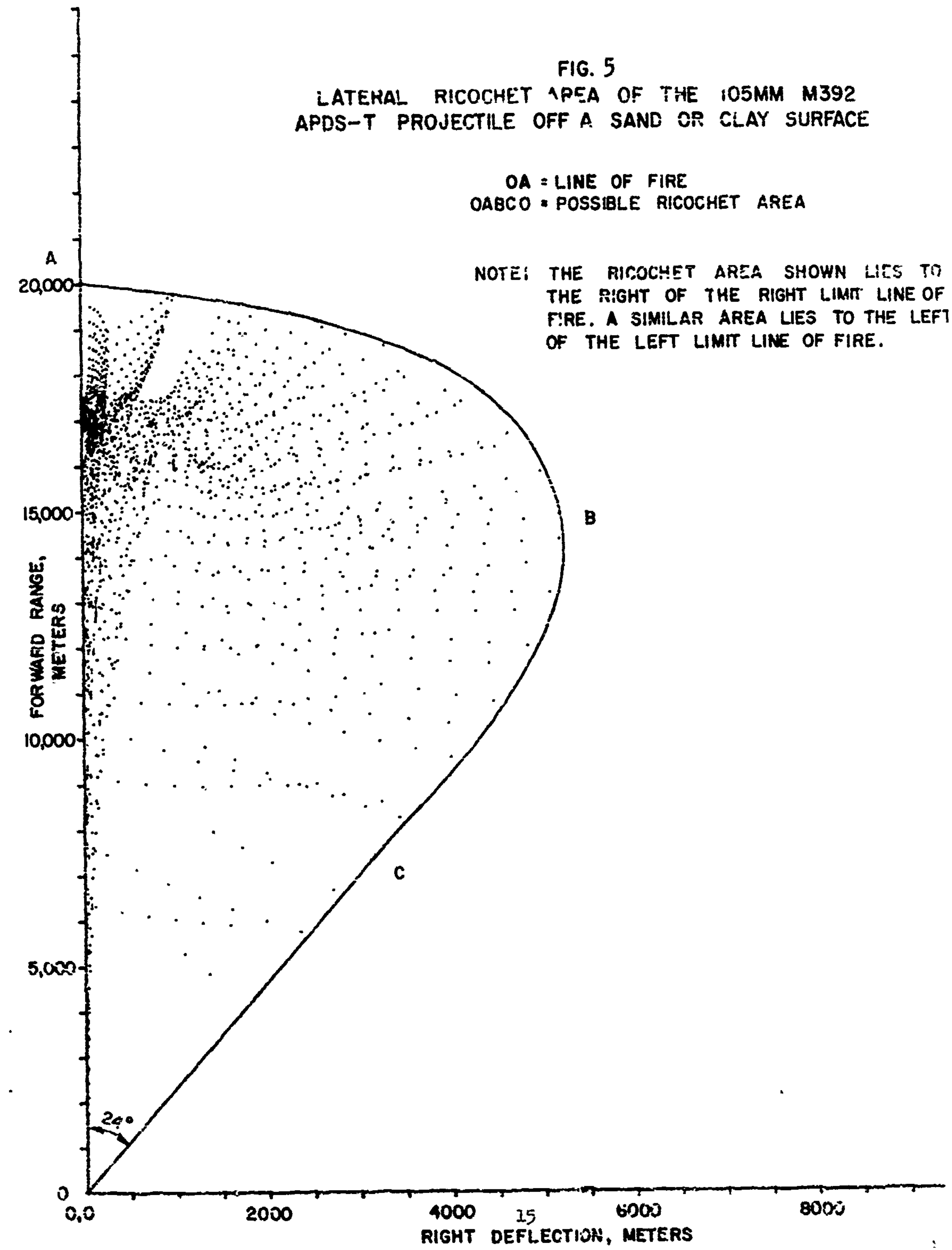

FIG. 5
LATERAL RICOCHET AREA OF THE 105MM M392 APDS-T PROJECTILE OFF A SAND OR CLAY SURFACE

V. REFERENCES

1. Birkhoff, G. Ricochet off Land Surfaces. BRL Report No. 535, March 1945.

2. Hitchcock, H. P. Effects of Ricochet on the Motion of Projectiles. BRL Report No. 629, February 1947.

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Range safety
Safety - Ricochet
Ricochet fire -
Danger area

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